# Questions: Direction-Sensitive Response in the Central Limit Theorem

The Central Limit Theorem (CLT) is usually introduced as a theorem about sums of independent random variables. Geometrically, however, it can be viewed as the response of a particular linear filter.

For an input vector $x$,

$$x = (x_1, x_2, \ldots, x_n),$$

the classical CLT considers the dot product

$$(1, 1, \ldots, 1) \cdot x,$$

or, more generally,

$$(\pm 1, \pm 1, \ldots, \pm 1) \cdot x,$$

where the signs may vary.

This raises some interesting questions.

- Does the CLT possess an implicit directional bias because it naturally emphasizes dense equal-weight directions?
- Are equal-weight sign vectors special because they maximize averaging over the input coordinates?
- How does the response change when the projection direction becomes sparse or has highly unequal coefficients?
- Is there a useful notion of **direction-dependent Gaussianization**?

For Gaussian inputs, every projection direction is statistically equivalent.

- The distribution of the projection depends only on its variance.
- Rotational symmetry means there is no preferred direction.
- Every unit vector produces the same one-dimensional Gaussian distribution.

For non-Gaussian product distributions (for example, independent uniform variables), the situation is different.

- Coordinate directions reproduce the original marginal distribution.
- Equal-weight sum or sum-and-difference directions produce distributions that are much closer to Gaussian.
- Different projection directions Gaussianize at different rates.

This suggests that the CLT may exhibit a form of **response anisotropy** when applied to non-Gaussian inputs.

Rather than simply asking whether a projection converges to a Gaussian, one could ask:

- Which projection directions converge fastest?
- Which converge slowest?
- How much directional variation exists for a given input distribution?
- Can this directional variation serve as a quantitative measure of non-Gaussian structure?

Viewed this way, the CLT is not merely a theorem about sums, but a theorem about the statistical response of particular families of linear filters. Comparing that response for Gaussian and non-Gaussian inputs may provide another way to study the geometry of high-dimensional probability distributions and the behavior of structured random projections.